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ANALYSIS OF A PRIORITY QUEUEING SYSTEM WITH PARTIAL BUFFER SHARING SCHEME AND ONE SERVER IN RESERVE

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Abstract:

This paper is concerned with analysis of a queueing system whose buffer is being shared partially by customers of two priority. The system has two servers and L waiting space, therefore there can be L+2 customers in the system including two in service. If the buffer occupancy becomes greater or equal to T, only high priority customers are allowed to enter the system. In this system, up to level K (K≥T) single server renders the services and second server is called up as buffer occupancy crosses the limit K. Then second server serves the system along with first server i.e. within limit 1 to K one server and K+1 to L two servers serve the system. Service times are identical for all type of customers and are assumed to be generally distributed.Customers of both priority class arrive according to Poisson distribution. A computational method is developed to compute system state probabilities in order to find other performance measures.

Key words- Priority queue, buffer, reserve server, threshold, laplace.

The main objective of this paper is to build a queueing model for optimal threshold and buffer size. In this model low priority customer can access the buffer if the buffer occupancy is less than a giver integral value T called the threshold. High priority and Low priority customers arrive in the system according to Poisson process with rates λ_1 and λ_2. Service times are assumed to be identical and generally distributed. Only one server renders service in the system if buffer occupancy is less than of equal to K and two servers serve if buffer occupancy is greater than K. Maximum buffer space is L (T≤ k≤L).We developed a computational method to compute state probabilities & loss probabilities.

This model has a number of practical application e.g. in a hospital which has a limited capacity, the event of an increase in the number of serious patients going beyond the limit another doctor needs to be called up.

There is a large literature on developing priority control scheme for efficient buffer utilization. Lee et. al. (8) developed a queueing model for optimal control of partial buffer sharing in ATM. For the same type of scheme a recursive method (9) is also proposed to Bernoulli arrival queues. Lio (10) analysed queueing of partial buffer sharing with Markov modulated Poisson inputs. Akyildiz (1) suggested two mechanism for controlling the loss priority: 'push-out mechanism' and capital buffer sharing mechanism'. There is also significant research in the area of multiserver queues. Escobar et al (2) found approxiamate solution for multiserver queues with Erlangian Service Times. Gupta et. al. (4) computed the steady state probabilities recursively of (M/G/1)/K queueing system. Mayhugh (11) obtained steady state solution of the queue M/E_k/r.

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Assumptions

λ_1 = arrival rate of high priority customer

λ_2 = arrival rate of low priority customer

λ = total arrival rate of customer ($\lambda_1+\lambda_2$)

T = threshold (low priority customers stop entering in the system if buffer occupancy is greater than or equal to T)

K = system level afterwards second server is called up.

L = buffer capacity

S(x) = service time distribution function.

s(x) = service time probability density function.

$S^*(\theta)$ = Laplace transform of s(x)

E(S) = mean service time

Var (S) = Variance of service time.

P_{LOSS1} = loss probability of high priority cuystomer.

P_{LOSS2} = loss probability of low priority customer.

R(t) = remaining service time of the customer in service at time t.

N(t) = system size at time t.

$P_0(t) = P_r[N(t)=0]$

$P_n(x,t)\Delta x = P_r[N(t)=n, x \le R(t) \le x+\Delta x]$, (n=1....K+1)

$P_0 = \underset{t\to\infty}{Lt} P_0(t)$

$P_n(x) = \underset{t\to\infty}{Lt} P_n(x,t)$

Steady-state system equations

Following are the steady state system equation:

When one server is rendering services in the system:

$$0 = -\lambda P_0 + P_1(0) \quad ...(1)$$

$$-\frac{d}{dx}P_1(x) = -\lambda P_1(x) + s(x)P_2(0) + \lambda P_0\, s(x) \quad ...(2)$$

$$-\frac{d}{dx}P_n(x) = -\lambda\, P_n(x) + s(x)P_{n+1}(0) + \lambda\, P_{n-1}, (n = 2,3...T) \quad ...(3)$$

$$-\frac{d}{dx}P_{T+1}(x) = -\lambda_1 P_{T+1}(x) + s(x)P_{T+2}(0) + \lambda P_T(x) \quad ...(4)$$

$$-\frac{d}{dx}P_n(x) = -\lambda_1 P_n(x) + s(x)P_{n+1}(0) + \lambda_1 P_{n-1}(x)(n = T+2....K) \quad ...(5)$$

When two servers are providing services in the system i.e.system size is greater than K.

$$-\frac{d}{dx}P_n(x) = -\lambda_1\, P_n(x) + 2s(x)\, P_{n+1}(x) + \lambda_1\, P_{n-1}(x)$$

(n=K+1...............L+1) ...(6)

$$-\frac{d}{dx}P_{L+2}(x) = -\lambda_1 P_{L+1}(x) \quad ...(7)$$

We define laplace transform as

$$P_n^*(\theta) = \int_0^\infty e^{-\theta x} P_n(x)\,dx \text{ for } 1 \le n \le L+1$$

Taking Laplace transform of equations (1) to (7)

$$(\lambda-\theta)P_1^*(\theta)=S^*(\theta)\left[\lambda P_0+P_2(0)\right]-P_1(0) \quad \ldots(8)$$

$$(\lambda-\theta)P_n^*(\theta)=\lambda P_{n-1}^*(\theta)+S^*(\theta)P_{n+1}(0)-P_n(0)\ (n=2.....T) \quad \ldots(9)$$

$$(\lambda_1-\theta)P_{T+1}^*(\theta)=\lambda P_T^*(\theta)+S^*(\theta)P_{T+2}(0)-P_{T+1}(0) \quad \ldots(10)$$

$$(\lambda_1-\theta)P_n^*(\theta)=\lambda_1 P_{n-1}^*(\theta)+S^*(\theta)P_{n+1}(0)-P_n(0)\ (n=T+2.......K) \quad \ldots(11)$$

$$(\lambda_1-\theta)P_n^*(\theta)=\lambda_1 P_{n-1}^*(\theta)+2S^*(\theta)-P_{n+1}(0)-P_n(0)\ (n=K+1.......L) \quad \ldots(12)$$

$$-\theta P_{L+2}^*(\theta)=\lambda_1 P_{L+1}^*(\theta)-P_{L+2}(0) \quad \ldots(13)$$

Calculation of state probabilities:

Let the steady state probabilities be

$$P_n=P_n^*(0)=\int_0^\infty P_n(x)dx,\ (n=1,2.....K,K+1,...L+2) \quad \ldots(14)$$

Adding equations (8) to (13)

$$-\theta\sum_{i=1}^{L+2} P_i^*(\theta)=S^*(\theta)\sum_{i=1}^{K+1} P_i(0)+2S^*(\theta)\sum_{i=K+2}^{L+2} P_i(0)-\sum_{i=1}^{L+2} P_i(0)$$

$$\Rightarrow \sum_{i=1}^{L+2} P_i^*(\theta)=\left[\frac{1-S^*(\theta)}{\theta}\right]\sum_{i=1}^{L+2} P_i(0)-\frac{S^*(\theta)}{\theta}\sum_{i=K+2}^{L+2} P_i(0) \quad \ldots(15)$$

As $\theta \to 0$

$$\sum_{i=1}^{L+2} P_i^*(\theta)=-S^{*(1)}(\theta)\sum_{i=1}^{L+2} P_i(0)-S^{*(1)}(\theta)\sum_{i=K+2}^{L+2} P_i(0),$$

{using L' hospital rule)

$$\Rightarrow \sum_{i=K+2}^{L+2} P_i^*(0)=E(S)\left[\sum_{i=1}^{L+2} P_i(0)+\sum_{i=K+2}^{L+2} P_i(0)\right] \quad \ldots(16)$$

Using (1) in (8) and assuming $\theta=\lambda$ and $\theta=0$ respectively

$$P_2(0)=\left[\frac{1-S^*(\lambda)}{S^*(\lambda)}\right]P_1(0) \quad \ldots(17)$$

and

$$P_1^*(0)=\frac{1}{\lambda}P_2(0) \quad \ldots(18)$$

Where S*(λ) Is the probability that no customerarrive during a service time.

Now letting $\theta=\lambda$ in equation (9), we get

$$P_{n+1}(0)=\frac{P_n(0)-\lambda P_{n-1}^*(\lambda)}{S^*(\lambda)},[n=2...T] \quad \ldots(19)$$

Now, we define j[th] derivative of $P_n^*(\theta)$ as

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$$\frac{d^j}{d\theta^j} P_n^*(\theta) = P_n^{*(j)}(\theta), \quad (n = 1.....L+2) \qquad ...(20)$$

From equation (8) & (9)

$$P_1^{*(j)}(\lambda) = -\frac{1}{j+1} S^{*(j+1)}(\lambda) \left[P_1(0) + P_2(0)\right](j = 0,....T-2) \qquad ...(21)$$

$$P_n^{*(j)}(\lambda) = -\frac{1}{j+1}\left[\lambda P_{n-1}^{*(j+1)}(\lambda) + P_{n+1}(0) S^{*(j+1)}(\lambda)\right]$$

$(n=2.....T, j=0, ... T-n)$ ...(22)

Hence $P_n(0)$ $3 \le n \le T+1$ can be obtained in terms of P_0 recursively from (21) and (22)

Now substituting θ=λ in equation (10) we get

$$P_{T+2}(0) = \frac{P_{T+1}(0) - \lambda P_T^*(\lambda_1)}{S^*(\lambda_1)} \qquad ...(23)$$

From equation (9) and (10) we have

$$P_1^*(\lambda_1) = \frac{S^*(\lambda_1)\left[P_1(0) + P_2(0)\right] - P_1(0)}{\lambda_2} \qquad ...(24)$$

$$P_n^*(\lambda_1) = \frac{\lambda P_{n-1}^*(\lambda_1) + S^*(\lambda_1) P_{n+1}(0) - P_n(0)}{\lambda_2} \qquad ...(25)$$

Now letting $\theta=\lambda_1$ in equation (11)

$$P_{n+1}(0) = \frac{P_n(0) - \lambda_1 P_{n-1}^*(\lambda_1)}{S^*(\lambda_1)}, \quad (n = T+2...... K) \qquad ...(26)$$

Again substitution θ=λ in equation (2)

$$P_{n+1}(0) = \frac{P_n(0) - \lambda_1 P_{n-1}^*(\lambda_1)}{2S^*(\lambda_1)}, \quad (n = K+1...... L+1) \qquad ...(27)$$

From equation (8), (9), (10), (11) and (12):

$$P_1^*(\lambda_1) = \frac{S^{*(j)}(\lambda_1)\left[P_1(0) + P_2(0)\right] + j P_1^{*(j-1)}(\lambda_1)}{\lambda_2}(j = 1....K-T) \qquad ...(28)$$

$$P_n^{*(j)}(\lambda_1) = \frac{\left[\lambda P_{n-1}^{*(j)}(\lambda_1) + S^{*(j)}(\lambda_1) P_{n+1}(0)\right] + j P_n^{*(j-1)}(\lambda_1)}{\lambda_2}(n = 2...T, j = 1....K-n-1) \qquad ...(29)$$

$$P_{T+1}^{*(j)}(\lambda_1) = -\frac{1}{j+1}\left[\lambda P_T^{*(j+1)}(\lambda_1) + S^{*(j+1)}(\lambda_1) P_{T+2}(0)\right], (j = 0.....K-T-2) \qquad ...(30)$$

$$P_n^{*(j)}(\lambda_1) = -\frac{1}{j+1}\left[\lambda_1 P_{n-1}^{*(j+1)}(\lambda_1) + S^{*(j+1)}(\lambda_1) P_{n+1}(0)\right], (n = T+2, K-1, j = 0,...k-n-1)$$

$$P_n^{*(j)}(\lambda_1) = -\frac{1}{j+1}\left[\lambda_1 P_{n-1}^{*(j+1)}(\lambda_1) + 2S^{*(j+1)}(\lambda_1) P_{n+1}(0)\right], (n = K...L-1, j = 0....L-n-1) \qquad ...(31)$$

...(32)

with the help of above equations $P_n(0)$ can be obtained in terms of P_0

Now putting θ=0in equation (9), (10), (11), and (12)

$$P_n^*(0) = \frac{\lambda P_{n-1}^*(0) + P_{n+1}(0) - P_n(0)}{\lambda}, \quad (n = 2.....T) \qquad ...(33)$$

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$$P^{*}_{T+1}(0)=\frac{\lambda P^{*}_{T}(0)+P_{T+2}(0)-P_{T+1}(0)}{\lambda_1}, \qquad \ldots(34)$$

$$P^{*}_{n}(0)=\frac{\lambda_1 P^{*}_{n-1}(0)+P_{n+1}(0)-P_{n}(0)}{\lambda_1}, \quad (n=T+2.....K) \qquad \ldots(35)$$

$$P^{*}_{n}(0)=\frac{\lambda_1 P^{*}_{n-1}(0)+2P_{n+1}(0)-P_{n}(0)}{\lambda_1}, \quad (n=K+1.....L+1) \qquad \ldots(36)$$

Approximating equations (33), (34), (35) and (36)

$$P^{*}_{n}(0)=\frac{P_{n+1}(0)}{\lambda}, \quad (n=2.....T) \qquad \ldots(37)$$

$$P^{*}_{n}(0)=\frac{P_{n+1}(0)}{\lambda_1}, \quad (n=T+1.....K) \qquad \ldots(38)$$

$$P^{*}_{n}(0)=\frac{2P_{n+1}(0)}{\lambda_1}, \quad (n=K+1.....L+1) \qquad \ldots(39)$$

For getting P^{*}_{L+1} we differnetiate equation (13)

$$-\theta P^{*(1)}_{L+1}(\theta)-P^{*}_{L+1}(\theta)=\lambda_1 P^{*(1)}_{L}(\theta)$$

Now substituting θ=0 $P^{*}_{L+1}(0)=-\ \lambda_1 P^{*(1)}_{L}(0)$ ...(40)

And for getting $P^{*(1)}_{L}(0)$ we differentiate (8) to (12)

Let θ=0, we obtain

$$P^{*(1)}_{1}(0)=\frac{\lambda S^{*(1)}(0)\left[P_1(0)+P_2(0)\right]+P_2(0)}{\lambda^2} \qquad \ldots(41)$$

$$P^{*(1)}_{n}(0)=\frac{\left[\lambda P^{*}_{n-1}(0)+S^{*(1)}(0)P_{n+1}(0)\right]}{\lambda}$$

$$+\frac{\left[\lambda P^{*}_{n-1}(0)+P_{n+1}(0)-P_{n}(0)\right]}{\lambda^2}, \quad (n=2.....T) \qquad \ldots(42)$$

$$P^{*(1)}_{T+1}(0)=\frac{\left[\lambda P^{*(1)}_{T}(0)+S^{*(1)}(0)P_{T+2}(0)\right]}{\lambda_1}+\frac{\lambda P^{*}_{T}(0)+P_{T+2}(0)-P_{T+1}(0)}{\lambda_1^2} \qquad \ldots(43)$$

$$P^{*(1)}_{n}(0)=\frac{\left[\lambda_1 P^{*(1)}_{n-1}(0)+S^{*(1)}(0)P_{n+1}(0)\right]}{\lambda_1}+\frac{\lambda_1 P^{*}_{n-1}(0)+P_{n+1}(0)-P_{n}(0)}{\lambda_1^2}$$

(n=T+2.........K) ...(44)

and

$$P^{*(1)}_{n}(0)=\left[\frac{\lambda_1 P^{*(1)}_{n-1}(0)+2S^{*(1)}(0)\ P_{n+1}(0)}{\lambda_1}\right]+\left[\frac{\lambda_1 P^{*}_{n-1}(0)+2P_{n+1}(0)-P_{n}(0)}{\lambda_1^2}\right]$$

(n=K+1.....L) ...(45)

Now using normalizing condition, the steady state probabilities of the system size at an arbitrary epoch can be computed.

$$P_0+\sum_{n=1}^{L+2} P^{*}_{n}(0)=1 \qquad \ldots(46)$$

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Hence, mean system size $L = \sum_{n=0}^{L+2} n.P_n$ ...(47)

mean queue size $L_q = \sum_{n=1}^{L+2} (n-1) P_n$ ...(48)

mean system sojourn time of an arbitrary customer is

$$\frac{L}{\lambda_1 (1 - P_{LOSS\,1}) + \lambda_2 (1 - P_{LOSS\,2})} \quad ...(49)$$

mean queue waiting time of an arbitrary customer is

$$\frac{L_q}{\lambda_1 (1 - P_{LOSS\,1}) + \lambda_2 (1 - P_{LOSS\,2})} \quad ...(50)$$

LOSS PROBABILITTY OF CUSTOMER:-

Loss probability of high priority customer $P_{LOSS\,1} = P_{L+2}$ ...(51)

Loss probability of low priority customer

$$P_{LOSS\,2} = \sum_{N=T+1}^{L+2} P_n \quad ...(52)$$

CONCLUSION

For the proposed model of priority queue with two types of customers and having two servers out of which one is reserved and called only when number of customers are greater than the threshold, we have derived expressions for mean queue size, mean system size. Loss probabilities of high priority customer and low priority customer are found to derive mean system sojourn time and mean queue waiting time of an arbitrary customer. All these performance measures play a critical role in designing the infrastructure of queueing system.

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